Aufgaben und Lösungen zur Eulerschen Zahl e

Vereinfachen/Umformen

a) $e^0 = 1$

b) $e^1 = e$

c) $e^x \cdot e^y = e^{x+y}$

d) $\frac{e^x}{e^y} = e^{x-y}$

e) $(e^x)^y = e^{x \cdot y} = (e^y)^x$

f) $e^{-x} = \frac{1}{e^x}$

g) $e^{-2} = \frac{1}{e^2}$

h) $e^{-0,5} = e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}}$

i) $e^2 \cdot e^{-3} = e^{2-3} = e^{-1} = \frac{1}{e}$

j) $\sqrt{e} \cdot \sqrt[3]{e^4} = e^{\frac{1}{2}} \cdot e^{\frac{4}{3}} = e^{\frac{1}{2}+\frac{4}{3}} = e^{\frac{3}{6}+\frac{8}{6}} = e^{\frac{11}{6}} = \sqrt[6]{e^{11}} = e \cdot \sqrt[6]{e^5}$

k) $\frac{\sqrt{e}-\frac{1}{\sqrt{e}}}{\sqrt{e}+\frac{1}{\sqrt{e}}} = \frac{\sqrt{e}-\frac{1}{\sqrt{e}}}{\sqrt{e}+\frac{1}{\sqrt{e}}} \cdot \frac{\sqrt{e}}{\sqrt{e}} = \frac{e-1}{e+1}$

l) $\frac{e^4-4}{e^2+2} - \frac{e^5-2e^3}{e^3} = \frac{(e^2-2)\cdot(e^2+2)}{e^2+2} - \frac{e^3\cdot(e^2-2)}{e^3} = e^2 - 2 - (e^2 - 2) = 0$

m) $(e^2 + \frac{1}{e^2})^2 - (e^2 - \frac{1}{e^2})^2 = (e^2)^2 + 2 \cdot e^2 \cdot \frac{1}{e^2} + \left(\frac{1}{e^2}\right)^2 - \left((e^2)^2 - 2 \cdot e^2 \cdot \frac{1}{e^2} + \left(\frac{1}{e^2}\right)^2\right) =$
$= e^4 + 2 + \frac{1}{e^4} - \left(e^4 - 2 + \frac{1}{e^4}\right) = e^4 + 2 + \frac{1}{e^4} - e^4 + 2 - \frac{1}{e^4} = 2 + 2 = 4$

n) e ntspannung

o) $4 \cdot \frac{e^x}{e^{2x}} - \frac{6}{e^x} + 2e^{-x} = 4 \cdot \frac{1}{e^x} - \frac{6}{e^x} + \frac{2}{e^x} = \frac{4-6+2}{e^x} = 0$

p) $\frac{e^x-e^{-x}}{1+e^{-x}} = \frac{e^{-x}\cdot(e^{2x}-1)}{e^{-x}\cdot(e^x+1)} = \frac{e^{2x}-1}{e^x+1} = \frac{(e^x)^2-1}{e^x+1} = \frac{(e^x-1)\cdot(e^x+1)}{e^x+1} = e^x - 1$

q) $\frac{e^{2t}-2te^t+t^2}{e^{2t}-t^2} = \frac{(e^t-t)^2}{(e^t-t)\cdot(e^t+t)} = \frac{e^t-t}{e^t+t}$

r) $\frac{e^{t+2}-k\cdot e^{t-1}}{k\cdot e^t-k^2\cdot e^{t-3}} = \frac{e^{t-1}\cdot(e^3-k)}{ke^{t-3}\cdot(e^3-k)} = \frac{e^{t-1}}{k\cdot e^{t-3}} = \frac{e^{t-1-(t-3)}}{k} = \frac{e^{t-1-t+3}}{k} = \frac{e^2}{k}$

s) $\frac{e^{2r}\cdot e^{2-2s}\cdot e^{t-r-5}}{e^{-4-r}\cdot e^{-2s-3}\cdot e^{2t-1}} = \frac{e^{2r+2-2s+t-r-5}}{e^{-4-r-2s-3+2t-1}} = \frac{e^{r-2s+t-3}}{e^{-r-2s+2t-8}} = e^{r-2s+t-3-(-r-2s+2t-8)} =$
$= e^{r-2s+t-3+r+2s-2t+8} = e^{2r-t+5} = \frac{e^{2r+5}}{e^t}$

t) $\frac{(e^x)^y}{e^x-e^y} \cdot \left(\frac{1}{e^x} - \frac{1}{e^y}\right) = \frac{e^{x\cdot y}}{e^x-e^y} \cdot \left(\frac{e^y}{e^x\cdot e^y} - \frac{e^x}{e^y\cdot e^x}\right) = \frac{e^{x\cdot y}}{e^x-e^y} \cdot \frac{e^y-e^x}{e^{x+y}} = -\frac{e^{x\cdot y}}{e^{x+y}}$

u) $\frac{\sqrt{e}-1}{\sqrt{e}+1} - \frac{e+1}{e-1} = \frac{\sqrt{e}-1}{\sqrt{e}+1} \cdot \frac{\sqrt{e}-1}{\sqrt{e}-1} - \frac{e+1}{e-1} = \frac{(\sqrt{e}-1)^2}{e-1} - \frac{e+1}{e-1} = \frac{e-2\sqrt{e}+1-(e+1)}{e-1} = \frac{e-2\sqrt{e}+1-e-1}{e-1} = -\frac{2\sqrt{e}}{e-1}$